

# Virtual photon impact factors with exact gluon kinematics

A.Bialas

M.Smoluchowski Institute of Physics  
Jagellonian University, Cracow\*  
Institute of Nuclear Physics, Cracow

H.Navelet and R. Peschanski  
Service Physique Theorique, CE Saclay†

February 1, 2001

## Abstract

An explicit analytic formula for the transverse and longitudinal impact factors  $S_{T,L}(N, \gamma)$  of the photon using  $k_T$  factorization with exact gluon kinematics is given. Applications to the QCD dipole model and the extraction of the unintegrated gluon structure function from data are proposed.

## 1 Introduction

In the present knowledge on perturbative QCD resummations at leading level in logarithms of the energy (i.e.  $\log 1/x_{Bj}$ ) the coupling of external sources, in particular a virtual gluon in deep-inelastic reactions, is based on the theorem of  $k_T$  factorization [1], proven in the leading logarithmic approximation of

---

\*Address: Reymonta 4, 30-059 Krakow, Poland; e-mail: bialas@th.if.uj.edu.pl

†Address: SPhT CEA-Saclay F-91191 Gif-sur-Yvette Cedex, France;  
emails: navelet@spht.saclay.cea.fr, pesch@spht.saclay.cea.fr

QCD [1]. The theorem states that the “unintegrated” gluon distribution, i.e. the distribution of energy *and* transverse momentum of gluons in the target, factorizes from the rest of the process. The remaining factor is the so-called “impact factor”. Consequently, this “impact factor” is an universal quantity, the same in all processes initiated by the same external source, e.g. the photon. The “unintegrated” gluon distribution will depend on the target, but again the target impact factor can also be factorized out, leaving place to an universal interaction term, given by the Balitsky, Fadin, Kuraev Lipatov BFKL Pomeron [2]. At next-to-leading level, the modified interaction term is now known [3] but not yet are the impact factors determined at this order of perturbation [4]. It is expected that the effect of the exact kinematics of the exchanged gluons, which is our subject, gives the main contribution to these higher order terms.

The  $k_T$  factorized impact factors can be conveniently expressed in terms of two Mellin variables,  $\gamma$ , conjugated to transverse momentum squared and  $N$ , conjugated to energy. Up to now, only the impact factors at  $N = 0$  were considered, since, strictly speaking,  $k_T$  factorization has been proven only at infinite energy, implying  $N = 0$ . If, however the validity of  $k_T$  factorization is extended to the case of exact gluon kinematics, it implies the knowledge of the combined  $\gamma, N$  dependence of impact factors. To our knowledge, this combined dependence has been derived only for real photoproduction of heavy flavors [5]. It is the purpose of the present paper to give an explicit analytic expression for the  $\gamma, N$  dependence of virtual photon impact factors. To this end, using the  $k_T$  factorization and exact gluon kinematics, we derive the explicit formulae for the total cross section of longitudinal and transverse photons on any target with a given distribution of gluons.

In the next section 2, the formulae for the (virtual) photon impact factors following from  $k_T$  factorization are given. The details of the calculation, implying multidimensional integration and resummation of generalized hypergeometric functions are presented in section 3 for the longitudinal case and in section 4 for the transverse one. Applications of our results are given in section 5.1 for the QCD dipole model [6] which turned out to be rather successful in description of the deep inelastic total and diffractive cross-section of the virtual photons [7]. In subsection 5.2 we suggest a model independent method of extraction from data of the unintegrated gluon structure function. Our conclusions are given in the last section.

## 2 An explicit formula for the total cross - sections using $k_T$ factorization

Our starting point is the formula for the total longitudinal and transverse photon cross-sections given in [8]:

$$\sigma_L \equiv \frac{4\pi^2\alpha}{Q^2} F_L = 4\alpha\alpha_s Q^2 \int_0^1 dz [z(1-z)]^2 \int \frac{dk^2}{k^4} \int d^2p \left( \frac{1}{p^2 + \hat{Q}^2} - \frac{1}{(p-k)^2 + \hat{Q}^2} \right)^2 g(x_g, k^2) . \quad (1)$$

Using

$$g(x_g, k^2) = \int \frac{dN}{2\pi i} (x_g)^{-N} \int \frac{d\gamma}{2\pi i} (k^2)^\gamma \tilde{g}_N(\gamma) , \quad (2)$$

we write

$$\sigma_L = \int \frac{dN}{2\pi i} (x_{Bj})^{-N} \int \frac{d\gamma}{2\pi i} \tilde{g}_N(\gamma) S_L(N, \gamma) (Q^2)^{\gamma-1} \quad (3)$$

with

$$S_L(N, \gamma) = 4\alpha\alpha_s \int_0^1 dz [z(1-z)]^2 \int dk^2 \left( \frac{k^2}{Q^2} \right)^{\gamma-2} \int d^2p \left( \frac{1}{p^2 + \hat{Q}^2} - \frac{1}{(\vec{p}-\vec{k})^2 + \hat{Q}^2} \right)^2 \frac{(\hat{Q}^2)^N}{[(\vec{p} - (1-z)\vec{k})^2 + \hat{Q}^2 + \hat{k}^2]^N} \quad (4)$$

where we have used the relation

$$x_g = x_{Bj} \frac{(\vec{p} - (1-z)\vec{k})^2 + \hat{Q}^2 + \hat{k}^2}{\hat{Q}^2}; \quad \hat{Q}^2 = z(1-z)Q^2 \quad (5)$$

Similarly,

$$\sigma_T = \frac{4\pi^2\alpha}{Q^2} F_T = \alpha\alpha_s \int_0^1 dz [z^2 + (1-z)^2] \int \frac{dk^2}{k^4} \int d^2p \left( \frac{\vec{p}}{p^2 + \hat{Q}^2} - \frac{\vec{p}-\vec{k}}{(\vec{p}-\vec{k})^2 + \hat{Q}^2} \right)^2 g(x_g, k^2) \quad (6)$$

and using (2) and (5) we have

$$\sigma_T \equiv \int \frac{dN}{2\pi i} (x_{Bj})^{-N} \int \frac{d\gamma}{2\pi i} \tilde{g}_N(\gamma) S_T(N, \gamma) (Q^2)^{\gamma-1} \quad (7)$$

with

$$S_T(N, \gamma) = \alpha \alpha_s \int_0^1 dz [z^2 + (1-z)^2] \int \frac{dk^2}{k^2} \left( \frac{k^2}{Q^2} \right)^{\gamma-1} \int d^2p \left( \frac{\vec{p}}{p^2 + \hat{Q}^2} - \frac{\vec{p} - \vec{k}}{(\vec{p} - \vec{k})^2 + \hat{Q}^2} \right)^2 \frac{(\hat{Q}^2)^N}{[(\vec{p} - (1-z)\vec{k})^2 + \hat{Q}^2 + \hat{k}^2]^N} . \quad (8)$$

It turns out that the integrals (4) and (8) can be explicitly performed and expressed in terms of  $\psi$  digamma functions. The details of the calculation are given in sections 3 and 4. Here we only quote the final formulae:

$$S_L(N, \gamma) = 8\alpha \alpha_s \frac{\pi \Gamma(\gamma + \delta + 1) \Gamma(\gamma + 1)}{\Gamma(N)} \frac{1}{(\delta^2 - 1)(\delta^2 - 4)} \left\{ \frac{\psi(\gamma + \delta) - \psi(\gamma)}{\delta} \times \frac{3N^2 - (\delta^2 - 1)}{2N} - 3 \right\} \quad (9)$$

$$S_T(N, \gamma) = 2\alpha \alpha_s \frac{\pi \Gamma(\gamma + \delta) \Gamma(\gamma)}{\Gamma(N)} \frac{1}{(\delta^2 - 1)(\delta^2 - 4)} \left\{ \frac{\psi(\gamma + \delta) - \psi(\gamma)}{\delta} \times \frac{N^2(3(N+1)^2 + 9) - 2N(\delta^2 - 1) + (\delta^2 - 1)(\delta^2 - 9)}{4N} - \frac{1}{2} (3(N+1)^2 + 9 + (\delta^2 - 1)) \right\} , \quad (10)$$

where we adopted the convenient notation  $\delta \equiv N - 2\gamma + 1$ ,

Note that the poles at  $\delta = 0, \pm 1, \pm 2$  in formulae (9,10) are actually absent due to zeroes in the numerators, as it should from regularity of the generalized (Meijer) hypergeometric functions appearing in the derivation, see later. This provides numerous and non trivial checks of the resummations leading to (9,10).

### 3 Longitudinal photon impact factor

As the first step in the calculation we observe that, using the symmetry of the integrand with respect to interchange of  $z$  and  $1 - z$ , the integrals in (4) can be written as a sum of two terms:

$$S_L(N, \gamma) = 8\alpha \alpha_s (Q^2)^{2-\gamma} (A - B) \quad (11)$$

where

$$A = \int_0^1 dz [z(1-z)]^2 \int \frac{dk^2}{k^4} (k^2)^\gamma \int d^2p \frac{1}{(p^2 + \hat{Q}^2)^2} \frac{(\hat{Q}^2)^N}{[(\vec{p} - (1-z)\vec{k})^2 + \hat{Q}^2 + \hat{k}^2]^N} \quad (12)$$

$$B = \int_0^1 dz [z(1-z)]^2 \int \frac{dk^2}{k^4} (k^2)^\gamma \int d^2p \frac{1}{[p^2 + \hat{Q}^2][(p-k)^2 + \hat{Q}^2]} \frac{(\hat{Q}^2)^N}{[(\vec{p} - (1-z)\vec{k})^2 + \hat{Q}^2 + \hat{k}^2]^N} . \quad (13)$$

Using several times the identity

$$\frac{1}{C^M} = \frac{1}{\Gamma(M)} \int_0^\infty dt t^{M-1} e^{-tC} \quad (14)$$

we transform (12) and (13) into

$$A = \frac{1}{\Gamma(N)} \int_0^1 dz [z(1-z)]^2 (\hat{Q}^2)^N \int \frac{dk^2}{k^4} (k^2)^\gamma \int v dv \int dt t^{N-1} \int d^2p \exp\{-v(p^2 + \hat{Q}^2) - t[(\vec{p} - (1-z)\vec{k})^2 + \hat{Q}^2 + \hat{k}^2]\} , \quad (15)$$

$$B = \frac{1}{\Gamma(N)} \int_0^1 dz [z(1-z)]^2 (\hat{Q}^2)^N \int \frac{dk^2}{k^4} (k^2)^\gamma \frac{1}{\Gamma(N)} \int dv \int dv' \int dt t^{N-1} \int d^2p e^{-v(p^2 + \hat{Q}^2) - v'((p-k)^2 + \hat{Q}^2) - t[(\vec{p} - (1-z)\vec{k})^2 + \hat{Q}^2 + \hat{k}^2]} \quad (16)$$

Thus the integration over  $d^2p$  reduces to a gaussian form and can be easily performed. After rescaling the variables  $v = ty, v' = ty'$  and substitution  $u = tk^2$  the integrals over  $du$  and  $dt$  factorize from the rest and can be explicitly evaluated. The final result of these operations reads

$$A = \frac{\pi \Gamma(\gamma-1) \Gamma(N-\gamma+2)}{\Gamma(N)} (Q^2)^{\gamma-2} \int_0^1 dz z^\gamma (1-z) \int \frac{y dy}{(1+y)^{N-2\gamma+4}} (y+z)^{1-\gamma} , \quad (17)$$

$$B = \frac{\pi\Gamma(N-\gamma+2)\Gamma(\gamma-1)}{\Gamma(N)}(Q^2)^{\gamma-2} \int_0^1 dz [z(1-z)]^\gamma \int \frac{dydy'[(y+z)(y'+1-z)]^{1-\gamma}}{(1+y+y')^{N-2\gamma+4}}. \quad (18)$$

To evaluate integrals in (17) one takes  $h = 1+y$ ,  $y+z = h[1-(1-z)/h]$ . Using the formula for the Gauss series

$$(1-x)^{-a} = \sum_{n=0}^{\infty} \frac{\Gamma(a+n)}{\Gamma(a)} \frac{x^n}{n!} \quad (19)$$

and picking the factor  $\Gamma(\gamma-1)$  in formula (18), the integral over  $dv$  leads to

$$\begin{aligned} \Gamma(\gamma-1) & \int \frac{ydy}{(1+y)^{N-2\gamma+4}} (y+z)^{1-\gamma} = \\ & = \sum \frac{\Gamma(\gamma-1+n)}{(N-\gamma+2+n)(N-\gamma+1+n)} \frac{(1-z)^n}{n!} \\ & \equiv {}_2G_1(\gamma-1, N-\gamma+1; N-\gamma+3; 1-z), \end{aligned} \quad (20)$$

where the Meijer function  ${}_2G_1(u, v; w; t) \equiv \Gamma(u)\Gamma(v)/\Gamma(w) {}_2F_1(u, v; w; t)$ .

Substituting this into (17) we can integrate over  $z$  to obtain

$$A = \frac{\pi\Gamma(N-\gamma+2)\Gamma(\gamma+1)}{\Gamma(N)}(Q^2)^{\gamma-2} T_A \quad (21)$$

where

$$T_A = {}_3G_2(2, \gamma-1, N-\gamma+1; \gamma+3, N-\gamma+3; 1). \quad (22)$$

To calculate the integrals over  $dy$  and  $dy'$  in (18) we first rescale the variables  $y, y' \rightarrow zy, (1-z)y'$ . Then we introduce  $h = 1+y$ ,  $h' = 1+y'$  and, finally  $\xi = h/h'$ . The last change implies  $1+y+y' \rightarrow 1+zy+(1-z)y' = h'[1-z(1-\xi)]$ . Using again (19) the integral over  $dydy'$  can be transformed into the series

$$\begin{aligned} & \int \frac{dydy'[(y+z)(y'+1-z)]^{1-\gamma}}{(1+y+y')^{N-2\gamma+4}} = \\ & = [z(1-z)]^{2-\gamma} \frac{\Gamma(N+2-\gamma)}{N\Gamma(N-2\gamma+4)} \sum_{n=0}^{\infty} \frac{\Gamma(N-2\gamma+4+n)}{\Gamma(N-\gamma+3+n)} (z^n + (1-z)^n) \\ & \equiv [z(1-z)]^{2-\gamma} \frac{\Gamma(N+2-\gamma)}{N\Gamma(N-2\gamma+4)} [{}_2G_1(1, N-2\gamma+4; N-\gamma+3; z) \\ & \quad + (z \rightarrow 1-z)] . \end{aligned} \quad (23)$$

Using known relations [9] on  ${}_2F_1$  functions, one writes also

$$\begin{aligned} & \int \frac{dy dy' [(y+z)(y'+1-z)]^{1-\gamma}}{(1+y+y')^{N-2\gamma+4}} = \\ & = z^{2-\gamma} \frac{1}{N\Gamma(\gamma-1)} \{ {}_2G_1(\gamma-1, N-\gamma+2; N-\gamma+3; z) + (z \rightarrow 1-z) \} . \end{aligned} \quad (24)$$

Introducing this into (18) we perform integration over  $z$  to obtain finally

$$B \equiv \frac{2\pi\Gamma(N-\gamma+2)\Gamma(\gamma+1)}{\Gamma(N)N} (Q^2)^{\gamma-2} T_B \quad (25)$$

where

$$T_B = (Q^2)^{\gamma-2} {}_3G_2(3, \gamma-1, N-\gamma+2; \gamma+4, N-\gamma+3; 1) . \quad (26)$$

It turns out that the series in (22) and (26) can be summed up and expressed in terms of the digamma functions. One uses known relations on Meijer functions at  $z=1$ , see ref.[10]. One writes

$$\begin{aligned} T_A & \equiv {}_3G_2(2, a, b; a+4, b+2; 1) = \\ & {}_3G_2(2, a, b; a+4, b+1; 1) - {}_3G_2(2, a, b+1; a+4, b+2; 1) = \\ & \frac{1}{3} [(b-1) {}_3G_2(1, a, b-1; a+3, b+1; 1) - b {}_3G_2(1, a, b; a+3, b+2; 1)] . \end{aligned} \quad (27)$$

and

$$T_B \equiv {}_3G_2(3, a, b+1; a+5, b+2; 1) = \frac{b(b-1)}{6} {}_3G_2(1, a, b-1; a+3, b+2; 1) , \quad (28)$$

using the abbreviations

$$a = \gamma - 1; \quad b = N + 1 - \gamma . \quad (29)$$

Using the generic formula for hypergeometric functions  ${}_3F_2(1, a, b; a+p+1, b+q+1; 1)$  [11], one writes:

$$\begin{aligned} & {}_3G_2(1, a, b; a+p+1, b+q+1; 1) \equiv \\ & (-1)^{p+1} \frac{\Gamma(p+q+1)}{\Gamma(p+1)\Gamma(q+1)} \frac{\Gamma(b-a-p)}{\Gamma(b-a+q+1)} (\psi(a) - \psi(b)) + \\ & \left[ (-1)^p \frac{\Gamma(b-a-p)}{\Gamma(1-a)\Gamma(q+1)} \sum_{k=0}^{p-1} \frac{1}{k-p} \frac{1}{\Gamma(k+1)} \frac{\Gamma(1+q+k)\Gamma(1-a-p+k)}{\Gamma(b-a+q-p+k+1)} \right. \\ & \quad \left. + \{a <-> b, p <-> q\} \right] . \end{aligned} \quad (30)$$

All in all, using for convenience the notations  $\delta \equiv b - a - 1$  and  $N \equiv b + a$ , we get

$$T_A - \frac{2T_B}{N} = \frac{1}{(\delta^2 - 1)(\delta^2 - 4)} \left( \frac{\psi(a+1+\delta) - \psi(a+1)}{\delta} \frac{3N^2 - (\delta^2 - 1)}{2N} - 3 \right). \quad (31)$$

## 4 Transverse photon impact factor

$\sigma_T$  can be evaluated along the similar lines as  $\sigma_L$ . Here we mark only the main differences. The key point is that  $\sigma_T$  can be evaluated using  $\sigma_L$ , which gives a noticeable simplification of the painful calculation.

Let us come back to the calculation of  $\sigma_L$ , starting with formula (11) and define

$$(DL) \equiv Q^2 A ; (NDL) \equiv Q^2 B . \quad (32)$$

Using (8) we write

$$S_T(\gamma, N) = 2\alpha\alpha_s((DT) - (NDT)) (Q^2)^{1-\gamma} \quad (33)$$

with

$$(DT) = \int_0^1 dz [z^2 + (1-z)^2] \int dk^2 (k^2)^{\gamma-2} \int d^2 p \frac{(\vec{p})^2}{(p^2 + \hat{Q}^2)^2} \frac{(\hat{Q}^2)^N}{[(\vec{p} - (1-z)\vec{k})^2 + \hat{Q}^2 + \hat{k}^2]^N} \quad (34)$$

$$(NDT) = \int_0^1 dz [z^2 + (1-z)^2] \int dk^2 (k^2)^{\gamma-2} \int d^2 p \frac{\vec{p}(\vec{p} - \vec{k})}{[p^2 + \hat{Q}^2][(p-k)^2 + \hat{Q}^2]} \frac{(\hat{Q}^2)^N}{[(\vec{p} - (1-z)\vec{k})^2 + \hat{Q}^2 + \hat{k}^2]^N} . \quad (35)$$

Let us rewrite the quantities of interest in the following form:

$$\begin{aligned} (DL) &\equiv \int_0^1 dz A_L(z) I(z) \\ (NDL) &\equiv \int_0^1 dz A_L(z) J(z) \\ (DT) &\equiv \int_0^1 dz A_T(z) [K(z) - I(z)] \\ (NDT) &\equiv \int_0^1 dz A_T(z) [K(z) - J(z) - \frac{1}{2}L(z)] , \end{aligned} \quad (36)$$



where

$$A_L(z) = z(1-z) ; A_T(z) = z^2 + (1-z)^2 = 1 - 2A_L(z) . \quad (37)$$

The integrals  $I, J, K, L$  are defined as follows:

$$I(z) = \int \frac{dk^2}{k^4} k^{2\gamma} \int d^2p \frac{\hat{Q}^2}{(p^2 + \hat{Q}^2)^2} \frac{(\hat{Q}^2)^N}{[(\vec{p} - (1-z)\vec{k})^2 + \hat{Q}^2 + \hat{k}^2]^N} , \quad (38)$$

$$J(z) = \int \frac{dk^2}{k^4} k^{2\gamma} \int d^2p \frac{\hat{Q}^2}{[p^2 + \hat{Q}^2][(p-k)^2 + \hat{Q}^2]} \frac{(\hat{Q}^2)^N}{[(\vec{p} - (1-z)\vec{k})^2 + \hat{Q}^2 + \hat{k}^2]^N} , \quad (39)$$

$$K(z) = \int \frac{dk^2}{k^4} k^{2\gamma} \int d^2p \frac{1}{(p^2 + \hat{Q}^2)} \frac{(\hat{Q}^2)^N}{[(\vec{p} - (1-z)\vec{k})^2 + \hat{Q}^2 + \hat{k}^2]^N} , \quad (40)$$

$$L(z) = \int \frac{dk^2}{k^4} k^{2\gamma} \int d^2p \frac{k^2}{[p^2 + \hat{Q}^2][(p-k)^2 + \hat{Q}^2]} \frac{(\hat{Q}^2)^N}{[(\vec{p} - (1-z)\vec{k})^2 + \hat{Q}^2 + \hat{k}^2]^N} . \quad (41)$$

Consequently, one reads for  $\sigma_L$  :

$$(DL) - (NDL) = \int_0^1 A_L(z) [I(z) - J(z)] . \quad (42)$$

The corresponding expression for  $\sigma_T$  shows a nice simplification, since the integrand  $K(z)$  cancels from the calculation:

$$(DT) - (NDT) = \int_0^1 A_T(z) [J(z) - I(z) + \frac{1}{2}L(z)] . \quad (43)$$

Using (37), we get

$$\begin{aligned} (DT) - (NDT) &= 2[(DL) - (NDL)] + \\ &+ \int_0^1 dz [J(z) - I(z)] + \frac{1}{2} \int_0^1 dz (1 - 2z(1-z)) L(z) . \end{aligned} \quad (44)$$

From formula (20), it is straightforward to obtain (in the  $a, b$  notations):

$$I(z) = Q^{2a} \frac{\Gamma(b+1)}{\Gamma(b+a)} z^a {}_2G_1(a, b; b+2; 1-z) , \quad (45)$$

where  ${}_2G_1(a, b; b+2; 1-z) \equiv \Gamma(a)\Gamma(b)/\Gamma(b+2) {}_2F_1(a, b; b+2; 1-z)$ .

After some straightforward transformations from formula (25), one writes

$$J(z) = Q^{2a} \frac{\Gamma(b+1)}{\Gamma(b+a+1)} z(1-z) [z^{a-1} {}_2G_1(a, b+1; b+2; 1-z) + (z \rightarrow 1-z)] , \quad (46)$$

$$L(z) = Q^{2a} \frac{\Gamma(b)}{\Gamma(b+a+1)} [z^a {}_2G_1(a+1, b; b+1; 1-z) + (z \rightarrow 1-z)] , \quad (47)$$

where  $L(z)$  is obtained from  $J(z)$  by suppressing the factor  $z(1-z)$  and replacing  $a \rightarrow a+1, b \rightarrow b-1$ , due to the numerator  $k^2$  instead of  $\hat{Q}^2$  for  $J(z)$  in formula (41).

Inserting expressions (45,46,47) in formula (44), one gets after integration over  $z$ :

$$\begin{aligned} (DT) - (NDT) &= 2[(DL) - (NDL)] + \frac{\Gamma(b)\Gamma(a+1)}{\Gamma(a+b+1)} Q^{2a} \\ &\times [{}_3G_2(1, a+1, b; a+2, b+1; 1) - ba {}_3G_2(1, a, b; a+2, b+2; 1) \\ &\quad - (b-1)(a+1) {}_3G_2(1, a+1, b-1; a+3, b+1; 1)] . \end{aligned} \quad (48)$$

All in all, using for convenience the notations  $\delta \equiv b-a$  and  $N \equiv b+a$ , one finally obtains

$$\begin{aligned} (DT) - (NDT) &= \frac{\Gamma(b)\Gamma(a+1)}{\Gamma(a+b)} Q^{2a} \frac{1}{(\delta^2-1)(\delta^2-4)} \\ &\left\{ \frac{\psi(a+1+\delta) - \psi(a+1)}{\delta} \times \frac{N^2(3(N+1)^2+9) - 2N(\delta^2-1) + (\delta^2-1)(\delta^2-9)}{4N} \right. \\ &\quad \left. - \frac{1}{2} (3(N+1)^2+9+(\delta^2-1)) \right\} \end{aligned} \quad (49)$$

## 5 Applications

### 5.1 Comparison with the dipole model

In a recent paper [12] we have shown that the dipole model is, strictly speaking, not compatible with the formulae obtained from the  $k_T$  factorization including the exact kinematics of the corresponding Feynman diagrams. The point is that the results from  $k_T$  factorization are non-diagonal in impact parameter space, contrary to the fundamental assumption of the dipole model.

Thus, it is worthwhile to derive the explicit modifications of the model due to the gluon kinematics.

The formulae of the previous section can now be compared with those obtained in the QCD dipole model, which amounts to consider [7] the impact factors at  $N = 0$ :

$$S_L^{dip}(\gamma) \equiv S_L(N=0, \gamma) = \frac{\pi^2 \alpha \alpha_s}{3} \frac{\gamma(1-\gamma)}{1-\frac{2}{3}\gamma} \frac{\Gamma^2(1-\gamma)\Gamma^2(\gamma)}{\Gamma(\frac{3}{2}-\gamma)\Gamma(\frac{3}{2}+\gamma)} \quad (50)$$

$$S_T^{dip}(\gamma) \equiv S_T(N=0, \gamma) = \frac{\pi^2 \alpha \alpha_s}{3} \frac{(1+\gamma)(1-\frac{1}{2}\gamma)}{1-\frac{2}{3}\gamma} \frac{\Gamma^2(1-\gamma)\Gamma^2(\gamma)}{\Gamma(\frac{3}{2}-\gamma)\Gamma(\frac{3}{2}+\gamma)} . \quad (51)$$

The knowledge of  $S_{L,T}(N, \gamma)$  allows one to take into account the modifications of the QCD dipole model due to exact gluon kinematics. Indeed, let us insert the BFKL pole in the formula for the unintegrated gluon structure function

$$\tilde{g}_N(\gamma) = \frac{\bar{v}(\gamma)w(\gamma)}{N - \bar{\alpha}\chi(\gamma)} (Q_0^2)^{-\gamma} ; \chi(\gamma) \equiv 2\psi(1) - \psi(\gamma) - \psi(1-\gamma) , \quad (52)$$

where  $\bar{\alpha}$  is the effective value of the strong coupling constant  $\bar{\alpha} = \frac{\alpha_s N_c}{\pi}$ , in the BFKL kernel,  $w(\gamma)$  is the Mellin transform of the probability to find a dipole in the target,  $Q_0$  sets the typical model scale of the unintegrated gluon structure function  $g(x_g, k^2)$ , (see formula (2)) and

$$\bar{v}(\gamma) = \frac{\bar{\alpha}}{2^{2\gamma}\gamma} \frac{\Gamma(1-\gamma)}{\Gamma(1+\gamma)} \quad (53)$$

is the Mellin transform of the probability to find a gluon in a dipole. In its final form, the modified QCD dipole model for longitudinal (3) and transverse (7) cross sections with full kinematics read:

$$\sigma_L = Q^{-2} \int \frac{d\gamma}{2\pi i} (x_{Bj})^{-\bar{\alpha}\chi(\gamma)} \bar{v}(\gamma)w(\gamma)S_L(N = \bar{\alpha}\chi(\gamma), \gamma) \left(\frac{Q^2}{Q_0^2}\right)^\gamma , \quad (54)$$

$$\sigma_T = Q^{-2} \int \frac{d\gamma}{2\pi i} (x_{Bj})^{-\bar{\alpha}\chi(\gamma)} \bar{v}(\gamma)w(\gamma)S_T(N = \bar{\alpha}\chi(\gamma), \gamma) \left(\frac{Q^2}{Q_0^2}\right)^\gamma . \quad (55)$$

Note that in these formulae, one has  $\delta \equiv N - 2\gamma + 1 = \bar{\alpha}\chi(\gamma) - 2\gamma + 1$ .

Formulae (54) and (55) give the explicit dependence of the cross-sections in the shift of the hard pomeron intercept.

We have compared numerical results (in the saddle-point approximation) from the Eqs (9) and (10) with those obtained from Eqs (50) and (51) in the range  $.01 > x > .0001$  and  $20\text{Gev}^2 < Q^2 < 160\text{Gev}^2$ . It turns out that they give a rather similar dependence on both  $x$  and  $Q^2$  (the deviations do not exceed 5%). Normalization changes, however: the cross-sections including the full gluon kinematics are by about factor 2 smaller than the ones obtained from the high-energy approximation. Note the interesting fact that the ratio  $R = \sigma_L/\sigma_T$  is affected: it increases by about 15%.

It is not very surprising that the dependence on kinematic variables does not substantially differ in the two approaches. Indeed, this dependence is mostly controlled by the position of the saddle point which is the same in the two formulae. The rather important change in normalization, however, could not have been easily guessed. In particular the ratio of normalizations in  $R = \sigma_L/\sigma_T$  is a quantity of experimental interest.

## 5.2 Method of extraction of the unintegrated gluon distribution

The knowledge of the impact factors  $S_L(N, \gamma)$  and  $S_T(N, \gamma)$  as explicit analytic functions of their two variables allows a model independent determination of the unintegrated gluon structure function from  $\sigma_L$  or  $\sigma_T$  or from the experimental observable  $F_2 \equiv \frac{Q^2}{4\pi^2\alpha}(\sigma_T + \sigma_L)$ .

Indeed, formulae (3,7) yield

$$\begin{aligned}\tilde{\sigma}_{T,L}(N, \gamma) &\equiv \int \frac{dx_{Bj}}{x_{Bj}} (x_{Bj})^N \int dQ^2 (Q^2)^{-\gamma} \sigma_{T,L}(x_{Bj}, Q^2) \\ &= \tilde{g}_N(\gamma) S_{T,L}(N, \gamma) .\end{aligned}\tag{56}$$

Hence, from formula (2) defining the unintegrated gluon structure function, one gets

$$g(x_g, k^2) = \int \frac{dN}{2\pi i} (x_g)^{-N} \int \frac{d\gamma}{2\pi i} (k^2)^\gamma \frac{\tilde{\sigma}_{T,L}(N, \gamma)}{S_{T,L}(N, \gamma)} .\tag{57}$$

This relation (57) has quite interesting features.

First, it allows a determination of the unintegrated gluon structure function from experiments. For instance one may consider the ratio  $(\tilde{\sigma}_T + \tilde{\sigma}_L)/(\sigma_T + \sigma_L)$  using data on  $F_2$ .

Second, the expected universality properties of  $g(x_g, k^2)$  from  $k_T$  factorization gives predictions for various processes e.g. the ratio  $R = \frac{F_L}{F_T}$ , the photoproduction and leptonproduction of heavy flavors, diffractive leptonproduction of vector mesons and any other process where  $k_T$  factorization applies.

Third, the applicability of this relation goes beyond a specific model like the QCD dipole one, provided the coupling to the virtual photon comes from the exchange of two off-mass shell gluons with exact kinematics.

These results comes from the assumption that  $k_T$  factorization is a valid approximation, e.g. the dominance of the two considered Feynman diagrams for the impact factors, when exact gluon kinematics can be considered. It remains to know to what extent higher order QCD contributions and non perturbative corrections may spoil this approximation.

## 6 Conclusions

Our formulae (9,10) give the exact two-variable dependence of the longitudinal and transverse impact factors of the photon in terms of the usual Mellin variables  $N, \gamma$ .  $N$  is the variable conjugated to  $x_{Bj}$ , while  $\gamma$  is conjugated to  $Q^2$ . In the particular framework of the QCD dipole model, it gives the modification taking into account the shift in  $N = \bar{\alpha}\chi(\gamma)$  of the BFKL singularity. This results mostly in a change of the normalization of the cross-section, whereas the dependence on kinematic variables predicted by the QCD dipole model is hardly affected. Note that the relative normalization in  $R = \frac{F_L}{F_T}$  is increased by 15%.

More generally, our result opens the way of a model-independent extraction of the universal unintegrated gluon structure function which appears in various processes, whenever  $k_T$  factorization can be applied.

### Acknowledgements

A.B. thanks the Service de Physique Theorique of Saclay for support and kind hospitality. This investigation was supported in part by the KBN Grant No 2 P03B 086 14 and by the Subsidium of Fundation for Polish Science 1/99.

## References

- [1] S.Catani, M.Ciafaloni, F.Hautmann, Nucl.Phys. B366 (1991) 135; Nucl.Phys. B29A (1992)182; J.C.Collins, R.K.Ellis, Nucl.Phys. B360 (1991) 3; E.M. Levin, M.G. Ryskin, Yu.M. Shabelski, A.G. Shuvaev, Sov.J.Nucl.Phys. 53 (1991) 657.
- [2] L.N.Lipatov, *Sov.J. Nucl.Phys.* **23** (1976) 642; V.S.Fadin, E.A.Kuraev and L.N.Lipatov, *Phys. lett.* **B60** (1975) 50; E.A. Kuraev, L.N. Lipatov and V.S. Fadin, *Sov.Phys. JETP* **44** (1976) 45, **45** (1977) 199; I.I.Balitsky and L.N.Lipatov, *Sov.J. Nucl.Phys.* **28** (1978) 822; L.N. Lipatov *Zh.Eksp.Teor.Fiz.* **90** (1986) 1536 (Eng. trans. *Sov.Phys. JETP* **63** (1986) 904).
- [3] V.S.Fadin and L.N.Lipatov, Phys.Lett. B429 (1998) 127; M.Ciafaloni, Phys.Lett. B429 (1998) 363; M.Ciafaloni and G.Camici, Phys.Lett. B430 (1998) 349.
- [4] The next-to-leading calculations are presently under way: J.Bartels, S.Gieseke and C.-F. Ciao, hep-ph/0009102; V. Fadin, D. Ivanov, M. Kotsky, hep-ph/0007119 and V. Fadin, J. Bartels, private communications.
- [5] S. Catani, M. Ciafaloni, F. Hautmann, Nucl.Phys. (Proc.Suppl.) 29A (1992)182. We were informed by M.Ciafaloni, that the virtual photon impact factors might have been discussed (but not published) some time ago by F.Hautmann.
- [6] L.L. Frankfurt and M.I. Strikman, Phys. Rep. 160 (1988) 235; A.H. Mueller, Nucl. Phys. B335 (1990) 115, Nucl. Phys. B415 (1994) 373. N.N. Nikolaev and B.G. Zakharov, Z. Phys. C49 (1991) 607, Phys. Lett. B332 (1994) 184, Z.Phys. C64 (1994) 631, C53 (1992) 331. A.H.Mueller and B.Patel, Nucl.Phys. B425 (1994) 471.
- [7] H.Navelet, R.Peschanski and C.Royon, Phys.Lett. B366 (1996) 329; H.Navelet, R.Peschanski, C.Royon and S.Wallon, Phys. Lett. B385 (1996) 357; A.Bialas, R.Peschanski and C.Royon, Phys.Rev. D57 (1998) 6899; S.Munier, R.Peschanski and C.Royon, Nucl.Phys. B354 (1998) 297.

- [8] J.Kwiecinski, A.Martin and A.Stasto, Phys.Rev. D56 (1997) 3991.
- [9] I.S.Gradstein and I.M.Ryzhik, Table of Integrals, Series and Products, Academic Press (1980).
- [10] A.Prudnikov, Y.Brychkov, O.Marichev, Integrals and Series, Vol.3 (Gordon and Breach Science Publishers,1986).
- [11] S.Munier and H.Navelet, Eur. Phys .J. C13 (2000)651. Note a misprint in formula (A.38): instead of  $\frac{\pi}{\sin\pi b}$ , read:  $\frac{\sin\pi b}{\pi}(-1)^p$ .
- [12] A.Bialas, H.Navelet and R.Peschanski, hep-ph/0009248, Nucl. Phys. B593 (2001) 438.